The relativity of paraconsistency in science: Some reflections on different contradictions in scientific contexts

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Is logic relative?

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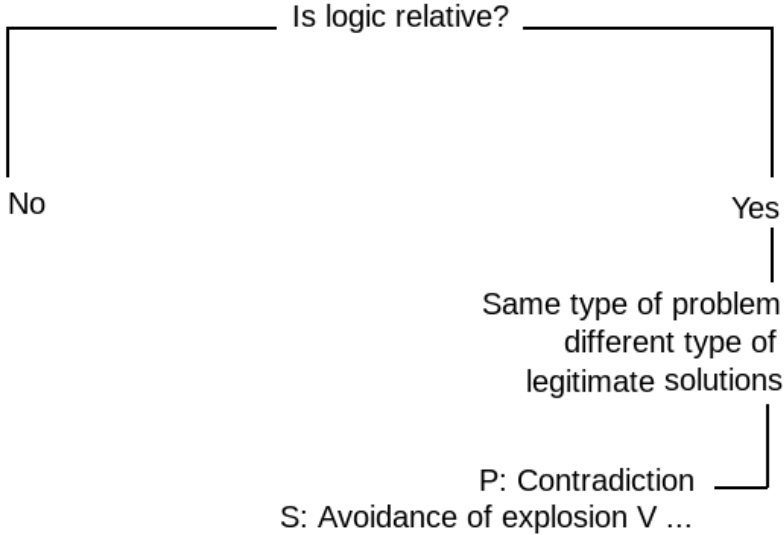
No,
there is a fixed set
of rules that
can/should be
applied universally.

Uniform solutions for the same
type of problem

P: Contradiction
S: Avoidance of
explosion
(paraconsistency)


Is logic relative?
No
Yes
Same type of problem
different type of
legitimate solutions
P: Contradiction
S: Avoidance of explosion V ...

Aim

The general claim: Logic (normative, human rationality) is relative.

The specific claim: Triviality is not a legitimate danger to scientific rationality when tolerating contradictions in the empirical sciences.

In order to do so, I appeal to the literature on risk analysis and claim that, if providing a risk analysis of the occurrence of triviality given practices of inconsistency toleration in the empirical sciences, one will notice that triviality is/should never be a legitimate concern.

Plan

1. Preliminaries
2. Triviality: a real challenge?
3. Risk
4. Risk and contradictions
5. Final remarks

Preliminaries

Contradiction

- A pair of propositions in which one is the negation of the other.

Principle of Explosion is one of the most characteristic principles of any explosive logic (including, of course, classical logic). It says that any theory if closed under an explosive logical consequence relation, will trivialize when containing a contradiction. A *contradiction* is a pair of propositions, where one is the negation of the other. A theory is *trivial* if any proposition is a theorem. Therefore, any inconsistent theory is trivial.

Preliminaries

Paraconsistent CR

- A logical consequence relation is *paraconsistent* if it does not validate PE; and a formal theory is paraconsistent if, despite containing a contradiction, it is not trivial.

Inconsistency toleration

- *Inconsistency toleration* is the phenomenon of working with inconsistent information and avoiding triviality at the same time.
  - In the case of human reasoning, inconsistency toleration demands a previous identification of a contradiction in the reasoning reasoning, as well as the capability of the agent to reason sensibly with the inconsistent information.

The intuition

- Logic: entailment V reasoning
  - Reasoning: Scientific reasoning looks like a good candidate (Scientific methodologies are some of the most sophisticated inferential filters, they 'clean' our reasoning about the phenomena that science explains...).
  - Entailment: Consequence relation of scientific theories.
- If logic is not relative, there is a fixed set of rules that can/should be applied universally. Particularly, for any occurrences of a given type of problem, solutions to a specific type of problem should be applied.

The hypotheses

Contradiction $\rightarrow$ Explosion/triviality

1. **Hyp. Classical:** If logic is not relative, for any case of inconsistency in science, PE is a legitimate danger and it cannot be avoided.
2. **Hyp. Paraconsistency:** If logic is not relative, for any case of inconsistency in science, PE is a legitimate danger and it should be avoided.
3. **Hyp. Non-PE:** PE is never a legitimate danger to human reasoning.

What would that mean that a theory/fragment of reasoning *is* inconsistent?

1. A proposition and its negation
   (Doxastical parity)
2. A justified(able) conjunction (Cf. Vickers 2013)

An example of the Hyp Paraconsistency

> *Examples of inconsistent but non-trivial theories are easy to produce. An example can be derived from the history of science. (In fact, many examples can be given from this area.) Consider Bohr's theory of the atom. According to this, an electron orbits the nucleus of the atom without radiating energy. However, according to Maxwell's equations, which formed an integral part of the theory, an electron which is accelerating in orbit must radiate energy. Hence Bohr's account of the behaviour of the atom was inconsistent. Yet, patently, not everything concerning the behavior of electrons was inferred from it, nor should it have been. Hence, whatever inference mechanism it was that underlay it, this* ***must have been paraconsistent****. (Priest et al. 2015: 2.1. My emphasis)*

Call this the *abductive argument (in favor of paraconsistency)*.

Scientific rationality in danger

There are three elements that are characteristic of scientific rationality in this sense, namely: (i) epistemic justification, (ii) context sensitivity and (iii) maximization of utility.

The dangers of contradictions

Contradictions entail everything: Acceptance requires belief, to accept a contradiction implies to believe a contradiction. But if rational belief is closed under entailment, because of the explosion principle, ”if someone believed a contradiction, they ought to believe everything, which is too much.” (Priest, 1998: 410). This objection has to components: on the one hand to believe everything clearly exceeds humans finite capabilities for managing information, and on the other hand, no sensible reasoning could take place once an agent has accepted a contradiction and, because of such acceptance, started believing everything. Thus, by a sort of *reductio*, epistemic agents, such as humans, cannot believe contradictions, and therefore, they cannot accept them.

The dangers of contradictions

Contradictions cannot be true: On the one hand, if the negation is understood as ”cancellation”:

> *¬A “cancels out” A. To assert A, and then to follow it by ¬A, is to cancel the first assertion by the second, and to end up saying nothing at all. The conjunction A & ¬A has no content. (Routley 1978: 395; quoted in McCall 2012: 444.)*

In addition, only meaningful statements can be true statements. For a sentence to be ’meaningful’ it is needed that it excludes something. Considering all the above, contradictions fail at doing so, and thus they are meaningless. For that reason, contradictions can never be true. On the other hand, in light of the Bar-Hillel-Carnap paradox, contradictions carry the maximum amount of information, this because

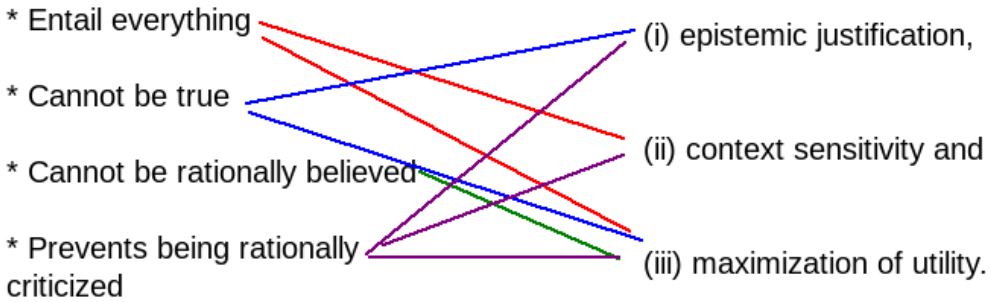
> *the less probable a statement is, the more informative it is, and so contradictions carry the maximum amount of information, and in the light of standard logic are, as a famous quote by Bar-Hillel and Carnap has it, “too informative to be true”. (Carnielli and Coniglio 2016: 2)*

Thus, **regardless if contradictions have no content at all or if they have all possible content, they can never be true.**

The dangers of contradictions

Contradictions cannot be believed rationally: since one ought to believe only what is true, contradictions ought not to be believed (Priest, 1998: 416). But even if contradictions could be true, because consistency is a constraint on rationality, they could never be rationally believed.

The acceptability of contradictions prevents people from being rationally criticized: "if you hold some view, and I object to it, there is nothing, rationally, to stop you from maintaining both your original view and my objection" (Priest 1998: 422). This is extremely problematic if one recognizes the possibility constant revision and evaluation of our beliefs as important features of rationality -specially if one also assumes that external criticism is a crucial motivator for revising our beliefs.


* Entail everything
* Cannot be true
* Cannot be rationally believed
* Prevents being rationally criticized
(i) epistemic justification,
(ii) context sensitivity and
(iii) maximization of utility.

We talk about the "danger"...

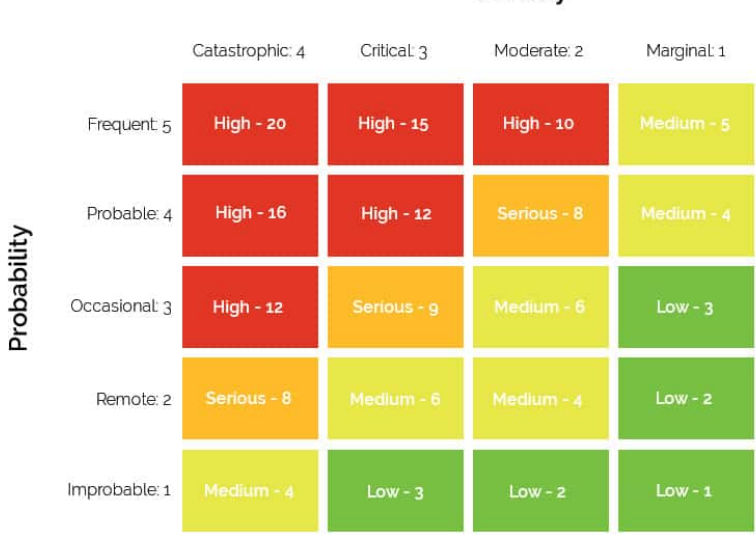
What does it mean *to be in danger*?

Risk

Across the literature, risk has been defined in many diverse ways; however, the majority of such characterizations understand 'risk' as a relational concept that comprises, at least, three elements:

$$\text{Risk}=(A,C,P)$$

- A: events,
- P: uncertainty (generally expressed in probabilistic terms) and
- C: consequences (usually expressed in terms of severity).


Severity
Catastrophic: 4
Critical: 3
Moderate: 2
Marginal: 1
Probability
Frequent: 5
High - 20
High - 15
High - 10
Medium - 5
Probable: 4
High - 16
High - 12
Serious - 8
Medium - 4
Occasional: 3
High - 12
Serious - 9
Medium - 6
Low - 3
Remote: 2
Serious - 8
Medium - 6
Medium - 4
Low - 2
Improbable: 1
Medium - 4
Low - 3
Low - 2
Low - 1

Risk (ii)

The different mechanisms that could be used to deal with the severity of the consequences of a particular event can be of two different types, (a) precaution-based and (b) risk-based.

- On the one hand, **precaution-based mechanisms** invest efforts in preventing the occurrence of the event that causes harm. The precaution-based (also called safety-preserving) mechanisms are the ways in which it is possible to avoid or minimize the severity of the consequences of a particular event before knowing the probability of such an event.

- On the other hand, the **risk-based approaches** invest efforts in mitigating the consequences of an event if the event were already obtained. Risk-management mechanisms are the different ways in which it is possible to avoid or minimize the severity of the consequences of a particular event once the probability of such an event is known.

Consequences of contradictions

1. **Basic scientific irrationality — the scientist:** A scientist believing a contradiction must be irrational (Davey, 2014). This is a particular instance of the objection 'Contradictions cannot be believed rationally',

   Distinctive object of harm: Epistemic justification for scientific rationality.

2. **False theories — theory:** "It is impossible for all the elements of a logically inconsistent set of sentences to be true, (. . .) a logically inconsistent theory is false" (Davey 2014; 3010). This is a particular instance of the objection 'Contradictions cannot be true'.

   Distinctive object of harm: Maximization of utility (the theory contains falsities).

- **Medium scientific irrationality— the scientist:** Preservation of justified belief through conjunction is valid, that is, the validity of the following is granted:

  JB_*SC* (A), JBSC ($\neg A$) $\vdash$ JB _*SC*($A\&\neg A$) This is a particular instance of the objections 'Contradictions entail everything' and 'Contradictions cannot be believed rationally'.

- **Evidential impasse — theory:** If a theory provides evidence in favour of A, but also provides evidence in favor of ¬A, that theory fails at solving problems in its discipline. This is a particular instance of the objections 'Contradictions cannot be true' previously presented

  Distinctive object of harm: Maximization of utility (the theory does not allow for efficient problem solving).

Consequences

- **Evidential impasse — scientist:** If a scientist believes to have conclusive evidence of A constituting a solution for Γ, but she also believes to have conclusive evidence of $\neg A$ constituting a solution for Γ, she will never be able to rationally choose between the two alternative solutions. This is a particular instance of the objections 'Contradictions cannot be true', 'Contradictions cannot be believed rationally', 'Contradictions entail everything' and 'the acceptability of contradictions prevents people from being rationally criticized'.

  Distinctive object of harm: Maximization of utility (the scientist cannot solve problems efficiently) and Epistemic justification (the scientist is never justified to take any of the mutually contradictory alternatives as a solution of the problem).

Consequences

- **Triviality — theory:**

  Distinctive object of harm: Scientific rationality —as the conjunction of (i) epistemic justification, (ii) context sensitivity and (iii) maximization of utility.

- T**riviality— scientist:** ”Contradictions cannot be true because if they were, we would end up sliding down into believing that everything is true (trivialism)” (Bueno, 2015: 466). This is a particular instance of the objections ’Contradictions entail everything’, ’contradictions cannot be rationally believed’ and ’the acceptability of contradictions prevents people from being rationally criticized’.

  Distinctive object of harm: Scientific rationality —as the conjunction of (i) epistemic justification, (ii) context sensitivity, and (iii) maximization of utility.

- Considering that, if scientific rationality were in danger because of the presence of contradictions, this would mean at least three main aspects of scientific rationality are about to be irremediably lost: (i) epistemic justification, (ii) context sensitivity and (iii) maximization of utility.
- Some of the general dangers that rationality, allegedly, faces when agents accept some contradictions are: (1) belief-triviality, (2) insufficiently explanatory valuations for contradictions, (3) believing a (logical) falsity and (4) impossibility of belief revision through criticism.

What about the risk?

Threat	Frequency	Object of harm	Effective-ness of threat conditiona-lized by:	Safety-preserving and Risk-management	Effectiveness of the threath
[1] Basic scientific irrationality	Medium Use of false information: Extremely high. Strong doxastic commitments towards the inconsistent set of information: Extremely Low.	Epistemic justification.	Rational belief closed under entailment. Valitity of principle of explosion Strong doxastic commitments towards the inconsistent set of information.	It is very likely that rational (in particular, justified) belief is not closed under entailment, nor closed under conjunction.t. (Sutton, 2007:68) Inconsistency toleration does not imply nor require strong doxastic commitments (such as JB) (See: Bueno 2017, Martínez-Ordaz 2017, Šešelja 2017)	Almost null. Scientists often do not expect to have justified belief in inconsistent sets of information. (Vickers 2013).

Threat	Frequency	Object of harm	Effectiveness of threat conditionalized by:	Safety-preserving and Risk-management	Effectiveness of the threath
[2] False theories	Extremely high (There is common agreement on the fact that our best scientific theories are only partially true. See for instance the literature on selective scientific realism: Chakravartty 2017; Lyons 2006; Saatsi 2017; Trizzio 2015; Vickers 2015,.)	Maximization of utility	The expectation of all the propositions contained in our scientific theories to be true. Contradictions do not have content.	The condition is (independently from the inconsistent science debates) understood to be mistaken.	Almost Null Scientists do not expect scientific theories to be fully true (scientific rationality seems to not be connected to such commitment).

[3] Medium scientific irrationality	Medium-Low	Epistemic justification	Threat [1]	Same as for Threat[1]	Almost null If justified belief obeys the laws of classical probability, the preservation of justified belief through conjunction is invalid.

[4] Evidential impasse theory	Low-extremely low It is common to obtain partial evidence for two non-compatible solutions. But, it is highly unlikely to obtain conclusive evidence for both solutions.	Maximization of utility	Threat [2]	Same as for Threat[2]	Evidence comes in degrees, and to have partial evidence in favour of two different things, does not lead to logical explosion.
[5] Evidential impasse Scientist	Extremely low When dealing with equal probabilities scientists look for additional evidence from other contexts (Putnam 1981), they don't remain in the impasse.	Maximization of utility and Epistemic justification	Threats [1], [3]. Rational choice closed under entailment. Contradictions have no content.	Same as the ones for threats [1], [2], [3], [4].	Almost null. Same as [4]

[6] Trivial theory	Null (there are cases of trivial theories from the formal sciences but so far, none from the empirical sciences.	Epistemic justification. Maximizing utilities. Weakening context sensitivity.	Threats [1], [2], [3], [4].	Paraconsistent underlying logics for the theory (Meheus 2002, Priest 2002, Batens 2017). Paraconsistent reasoning strategies (Brown & Priest 2004, 2015). Triviality is a formal phenomenon, use of mechanisms of content driven control (Vickers, 2013)	Not fully effective For the case of Frege's set theory, mathema-thicians still could propose rational alternatives to solve the problem linked to the contradiction in the theory. When a theory is trivial, context sensitivity weakens but doesn't go away. Scientists tend to shift to safer context to try to fix the trivial theory.

[7] Epistemic triviality	(there are cases of trivial theories from the formal sciences but so far, none from the empirical sciences.	Context sensitivity (if context sensitivity is doomed, scientific rationality will never recover)	Threats [1], [2], [3], [4], [5], [6].	Relevant (paraconsistent) underlying logics for fragments of human reasoning. Natural procedures for handling inconsistent information, such as, to break it up into consistent fragments, and then to operate within these. (Brown & Priest, 2004) Additional formal tools for representing inconsistent reasoning (standard non-adjunctive logics share the mechanisms mentioned above).	Null.

There are different ways to deal with contradictions...
and for many of them, explosion is not a relevant problem.

As we have learned from experience, the statistical frequency of seriously damaging contradictions in practice is very low and the level of harm that contradictions could pose against scientific rationality might be very low as well... Yet, comparing levels of risk of each of the alleged dangers that contradictions pose against of scientific rationality might be of use when philosophically fearing logical explosion in cases of empirical inconsistent science.

Final remarks

Contradiction $\rightarrow$ Explosion/triviality

1. **Hyp. Classical:** If logic is not relative, for any case of inconsistency in science, PE is a legitimate danger and it cannot be avoided.
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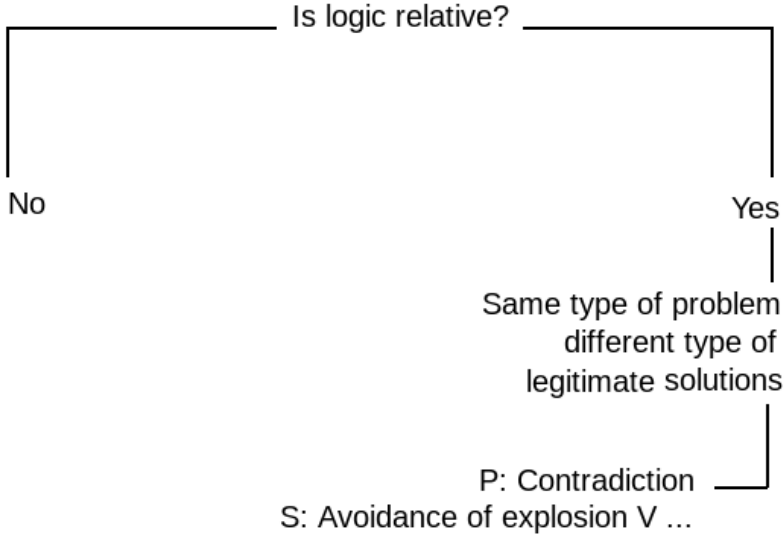
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