A methodological shift in favor of (some) paraconsistency in the sciences

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Abstract

Many have contended that non-classical logicians have failed at providing evidence of paraconsistent logics being applicable in cases of inconsistency toleration in the sciences. With this in mind, my main concern here is methodological. I aim at addressing the question of how should we study and explain cases of inconsistent science, using paraconsistent tools, without ruining into the most common methodological mistakes. My response is divided into two main parts: first, I provide some methodological guidance on how to approach cases of inconsistent science; and second, I focus on a peculiar type of formal methodologies for the scrutiny of inconsistent reasoning, the *Paraconsistent Alternative Approach* (henceforth, PAA) and argue that PAA can enhance a more accurate understanding of sensible reasoning in inconsistent contexts.

Keywords: Inconsistency toleration, paraconsistent logics, paraconsistent reasoning strategies, contradiction, human reasoning, ignorance.

Cite the published version:
Martínez-Ordaz, M.d.R. A Methodological Shift in Favor of (Some) Paraconsistency in the Sciences. *Log. Univers.* (2022).

This paper was awarded the 'Mexican Academy of Logic (AML) Logic Prize 2021'.
This prize is part of the project *A Prize of Logic in Every Country.*

1 Introduction

Principle of Explosion (PE) says that any (explosive) theory trivializes when containing a contradiction. A *contradiction* consists of a pair of propositions, where one is the negation of the other. A theory is *trivial* if it is possible

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to derive any proposition from it. Any inconsistent (explosive) theory will be trivial. A logical consequence relation is *paraconsistent* if it does not validate PE; and a formal theory is paraconsistent if, despite containing a contradiction, it is not trivial. *Inconsistency toleration* is the phenomenon of working with inconsistent information and avoiding triviality at the same time.[1]

The detection and the analysis of contradictions in human reasoning have a long history in Western philosophy. One of the main reasons that have motivated the analysis of contradictions lies on the assumption that the study of contradictory beliefs, and the different ways in which we manage (and resolve) such contradictions, play a fundamental role in elucidating the foundations of rationality (Cf. Rovane 2004). Furthermore, because science is often taken as a great exemplar of human rationality, the study of the role of contradictions in scientific reasoning is expected to be remarkably revealing in this sense.

That said, an important question in this respect is whether science could be inconsistent and non-explosive at the same time; this is, *can science be inconsistency tolerant?* On the one hand, if science could, in fact, be tolerant towards contradictions, this would provide important evidence in favor of the value of (some) paraconsistent logics for the scrutiny of science −and human rationality, in general. On the other hand, if science could be unproblematically inconsistent, this would give philosophers of science new methodological considerations for the evaluation, reconstruction and explanation of the scientific enterprise. And while the project of searching for instances of inconsistency toleration in the sciences is clearly promising for both philosophy of logic and philosophy of science, it has not prospered as expected.

Many have blamed this lack success on three methodological missteps: the (unjustified) presumption of logical explosion, the (unjustified) presumption of logical closure of scientific reasoning, as well as the underlying idea that the correct analysis of non-explosive inconsistent reasoning would privilege a particular (type of) paraconsistent logic. According to this explanation, logicians often lose sight of the original goal and end up exclusively "proposing alternative logics that *might* lurk in the background of scientific reasoning" (Cf. Brown and Priest 2015: 299. My emphasis); focusing their attention on their specific preferred logics and neglecting all evidence that might conflict with their philosophical commitments (Cf. Vickers 2013, Chap. 8).

My main concern here is methodological. I aim at addressing the question of how should we study and explain cases of inconsistent science without ruining into the same methodological mistakes. My response to this question is divided into two main parts: first, I provide some methodological guidance on how to approach cases of inconsistent science; and second, I focus on a peculiar type of formal methodologies for the scrutiny of inconsistent reasoning, the *Paraconsistent Alternative Approach* (henceforth, PAA) and argue that PAA can enhance a more accurate understanding of sensible reasoning in inconsistent contexts.

[1] In the case of human reasoning, inconsistency toleration demands a previous identification of a contradiction in the reasoning reasoning, as well as the capability of the agent to reason sensibly with the inconsistent information.

In order to do the above, I proceed as follows. In Sec. 2, I explain the philosophical relevance of analyzing cases of inconsistency toleration in scientific contexts. To do so, I adopt an anti-exceptionalist view about logic within which I frame some of the methodological errors of the traditional projects for the study of inconsistency in science. In Sec. 3, I suggest a way to modify the traditional methodology used when tackling non-explosive inconsistent reasoning. In Sec. 4, I discuss two formal approaches to inconsistent reasoning: the traditional one, logic-driven, and the PAA, strategy-guided; here, I also defend that the PAA can account for inconsistent scientific reasoning more successfully than traditional approaches. In Sec. 5, I sketch the relation between PAA and logical pluralism. Finally, in Sec. 6, I draw some conclusions.

2 What is at stake?

The main aim of this section is to discuss the philosophical value of identifying instances of inconsistency toleration in the sciences, as well as some methodological considerations for the scrutiny of cases of inconsistency toleration. The section is divided in two parts. The first one focuses on discussing the philosophical value of gathering independent evidence in favor of inferential phenomena such as inconsistency toleration. The second part discusses the problems of the so-called *abductive argument* in favor of paraconsistency (Cf. Michael 2016).

2.1 The value of evidence

Exceptionalism about logic is the view according to which the truths and the methodology of logic are significantly different from the ones of other scientific disciplines, making the study of logic an enterprise of a unique kind. In contrast, *anti-exceptionalism* is the view according to which logic is continuous with other sciences, in particular, with empirical disciplines (Cf. Quine and Ullian 1970; Quine 1986; Maddy 2002; Williamson 2007, 2013, 2015; Priest 2014).

Similarly, the applicability of a theory blocks an interpretative-type of *ad hoc*ness, as it grounds the ways in which the content of the theory can be interpreted in that particular case. Finally, applicability often broads the descriptive and explanatory scope of theories facilitating the resolution of relevant problems in the domain of application −preserving and increasing the fruitfulness of the theory in question.

> Given any theory, in science, metaphysics, ethics, logic, or anything else, we choose the theory which best meets those criteria which determine a good theory. Principal amongst these is adequacy to the data for which the theory is meant to account. In the present case, these are those particular inferences that strike us as correct or incorrect (...) Adequacy to the data is only one criterion, however. Others that are frequently invoked are: simplicity, non-(ad hocness), unifying power, fruitfulness. (Priest 2014: 217)

In the empirical sciences, the effective application of a particular theory within a specific domain is considered to be indicative of the (partial) success of the theory in question. This mainly because the applicability of any theory, often, requires the previous satisfaction of, at least, three of the main criteria alluded by Priest: adequacy, non-ad hocness and fruitfulness. For a theory to be satisfactorily applied within a domain, it has to have a previous important degree of adequacy to the data about that domain (Cf. Bueno 1997). In addition, when a theory is satisfactorily applied within a domain, the specific characteristics of this domain would constrain, limit and inform the inference patterns that hold within the theory in that particular case, preventing an inferential-type of ad hocness. Similarly, the applicability of a theory blocks an interpretative-type of ad hocness, as it grounds the ways in which the content of the theory can be interpreted in that particular case. Finally, applicability often broads the descriptive and explanatory scope of theories facilitating the resolution of relevant problems in the domain of application −preserving and increasing the fruitfulness of the theory in question.

The above considered, it should not come as a surprise that, at least, for the anti-exceptionalist, the epistemic benefits linked to applicability are not restricted to the empirical disciplines, but can be extended to the logical realm. Identifying a phenomenon that can be addressed in a meaningful way from a particular (type of) logic, counts as evidence in favor of the legitimacy of such a logic −this, in the same way in which the empirical adequacy of empirical theories warrants a significant connection between the theory and the independent world.

Furthermore, in the same way in which applicability counts as an telling pointer in scientific theory choice, it plays a crucial role for the same purposes in philosophical logic. As a matter of fact, large part of philosophical debates about logic relies on our capability to identify successful applications of certain theories and assessing their philosophical import. When a logical theory is applied to a novel domain, this is taken to be evidence of its strength and legitimacy (Cf. Hjortland 2017).[2]

When one identifies a phenomenon that cannot be explained without a specific logical theory, this counts as evidence in favor the theory's legitimacy −suggesting that the theory is more than a mere combinatorial exercise and that it can play an significant representational role. It is important to emphasize the connection between this attitude of gathering and evaluating evidence in favor of certain logics and adopting an anti-exceptionalist methodology. The idea behind this relation is that, when logicians seek for domains of application for their logical theories, and expect the applicability of their theories to be informative about their legitimacy and to play a role in processes of theory choice, what their are being committed to is the methodological criterion according to which evidence plays a privileged role in justifying our rational commitments

[2]Some of the philosophical debates that have been significantly enriched by the discussions about applications of specific logical theories include the long lasting discussion on logical pluralism versus logical monism (Cf. Williamson 2007, 2013, 2015; Priest 2014), and the one between exceptionalism and anti-exceptionalism about logic (Cf. Hjortland 2017).

towards specific theories −regardless of these theories being scientific or logical.[3]

> It is natural to think that logical theories are justified, in part, by the available evidence (...) For most scientific theories the observational data forms the bulk of the evidence (...) The evidence for a logical theory can come from a number of sources: from intuitions about validity or alethic modality, from mathematical theories and practice, from psychology of reasoning, from epistemic norms of rationality, and so on. (Hjortland 2017: 645)

For the case of paraconsistent logics, this independent evidence has come from two main sources: the study of paradoxes and the scrutiny of cases of inconsistent non-trivial human reasoning. In what follows, I focus only on the type of evidence that is expected to be obtained from the latter.

2.2 The fallacious abductive leap

When looking at the historical record, a large majority of paraconsistent logicians have recognized different instances of inconsistency toleration in the sciences. As a matter of fact, for many decades, philosophical research programs of paraconsistent logics, been concerned with analyzing the phenomenon of inconsistency toleration at the intersection of the levels of specific theories and the reasoning of particular epistemic agents. Some of the most famous examples of inconsistency toleration in the sciences include: Aristotle's theory of motion, the Early Calculus, Bohr's theory of the atom, and Classical Electrodynamics.

However, according to [Michael 2016], this 'evidence' in favor of paraconsistency is nothing but apparent, and it is obtained by a biased abductive argument that leads to interpret the relation between what the historical record has shown and the success of paraconsistent logics. "The idea here would be that there are interesting and productive inconsistent theories from which people do not infer random and unconnected conclusions; so, it might be thought, the logic they use does not licence such inferences" (Michael 2016: 3356).

> Examples of inconsistent but non-trivial theories are easy to produce.An example can be derived from the history of science. (In fact, many examples can be given from this area.) Consider Bohr's theory of the atom. According to this, an electron orbits the nucleus of the atom without radiating energy. However, according to Maxwell's equations,which formed an integral part of the theory, an electron which is accelerating in orbit must radiate energy.Hence Bohr's account of the behaviour of the atom was inconsistent. Yet, patently, not everything concerning the behavior of electrons was inferred from it, nor should it have been. *Hence, whatever inference mechanism it was that underlay it, this must have been paraconsistent.* (Priest et al. 2015: 2.1. My emphasis)

[3]This is a crucial point that is almost never made explicit in the corresponding literature but that is necessary for understanding the value of certain case studies in philosophical logic.

Call this the *abductive argument (in favor of paraconsistency)*.

This argument goes from a particular case that illustrates (temporal or alleged) inconsistency toleration to, by abduction, inferring that the best explanation of this toleration is that either the reasoning of the scientists or the theory in question were closed under a paraconsistent logical consequence relation. Once this conclusion is reached, it seems that the only remaining task is to determine which is the specific logic that allows the reasoning to be paraconsistent in the particular case.

There are two main methodological missteps that are followed in this argument: the presumption of logical explosion and the presumption of logical closure.

> **The presumption of logical explosion:** When the abductive argument concludes that, in cases of inconsistency toleration, the absence of logical triviality implies that the underlying reasoning *should* be paraconsistent, it also reveals that triviality, in the case of inconsistency, is not only expected as a possibility, but as the most likely and revealing outcome. [4]
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> The problem with this is that it also reveals that the question that the abductive argument is addressing is not the one of the logical constraints of inconsistency toleration, but a question of which paraconsistent logic could benefit from taking a particular case as evidence of its applicability. In this sense, this "argument turns on what people in fact infer or should infer, so builds in a normative aspect. The first question which needs to be raised concerns the status of the 'should' in the claim about whether they should have inferred random sentences from the theory" (Michael 2016: 3357).
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> This presumption has two important downfalls. First, it weakens the empirical (historical) adequacy of the explanations that (paraconsistent) logicians might provide regarding the broad phenomenon of inconsistency toleration -in human reasoning. Second, from a methodological point of view, it exhibits a dangerous bias that undermines the identification of domains for application of paraconsistent logics in scientific contexts.
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> **The presumption of logical closure:** This presumption is what grounds, at least partially, the previous one. If logical triviality represents a real threat against reasoning in sciences, this is caused by our expectations regarding (i) human rationality having to obey certain logical principles, as well as (ii) the existence of a logical closure that can determine the scope of such principles whitin human reasoning.
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> This presumption is seen as a methodological error is mainly because we have no evidence in favor of human rationality being constrained by the

[4] A key problem with this presumption is that triviality is the most likely outcome only in the case in which the underlying logic would be classical, but for the high likelihood of this being the case, we have no evidence. And if there is not enough evidence in favor of triviality being the most likely outcome, one should weaken one's expectations about how telling the absence of logical triviality might really be –either in favor or against any paraconsistent logic.

> rigor needed for (i) and (ii) to be the case. As a matter of fact, what psychologists and epistemologists have recognized is that epistemic agents respond to inferential challenges, such as inconsistencies, using pragmatic reasoning strategies that help them to solve particular problems in specific contexts and that do not privilege determined theories of inference (Cf. Harman 1984: 108, Michael 2016: 3355-57).

An even more severe methodological vice is revealed through the combination of these two unjustified presumptions:

> **The fallacious foundation:** The abductive argument relies on one implicitly accepting that the results of a correct analysis of inconsistent scientific reasoning would necessarily privilege a particular (paraconsistent) logic. Yet, this methodological choice is not only unjustified but it is also the source of the later, also unjustified, requirements of human reasoning being closed under a logical consequence relation and the absence of logical triviality being the most telling outcome of the constraints of the reasoning carried out in inconsistent contexts. This can be characterized as a sort of methodological *petitio principii*.

The main problem with the methodological missteps of the abductive argument is that, from the very beginning, the argument requires to overlook the salient features of inconsistent scientific reasoning. On the one hand, it neglects that the logical constraints of human reasoning, if any, would be of the shape of pragmatic and contextual reasoning strategies more than of logical principles (Cf. Harman 1984). On the other hand, the assumption of logical triviality (or lack thereof) being the most telling outcome of inconsistent reasoning fails at acknowledging the particularities of each case and their role in the handling of the corresponding contradictions –it also assumes that all cases of inconsistency toleration are, at least, structurally uniform, meaning that, the failure at handling a contradiction in one case would look the same as in any other case.

At this point, it should be clear to the reader that the problems of the abductive argument surpass the weak inferential link between the premises and the conclusion; as a matter of fact, they also undermine the quality of the evidence, if any, that could have been provided through it.

The initial data used in each instance of the abductive argument is expected to illustrate the explicit acknowledgment of a contradiction and, despite this, the preservation of sensible reasoning when working with such an information. However, this by itself only says that epistemic agents who might consider contradictions to be malign, found a way to not reject an inconsistent set of information and kept working with it. However, while this could be considered to be independent evidence in favor of a sort of inconsistency toleration, this is not yet evidence in favor of any particular logical procedure. This is where the fallacious abductive leap gets even more serious, when, in order to favor a conclusion of a logical nature, the analysis neglects a large amount of the historical relevant data that was expected to be crucial evidence regarding the applicability of specific logics.

In the next section, I briefly delve into the importance of finding a balance between historical/empirical and logical analyses of inconsistency toleration in the sciences.

3 A methodological shift

I introduce a methodology for the study of cases of inconsistency toleration in the sciences. The section is divided in two parts: the first one introduces some preliminaries on the different ways to approach historical evidence of inconsistency toleration. The second part focuses on sketching a methodology that avoids problems such as the ones of the abductive argument.

3.1 The paraconsistency landscape

According to many paraconsistent logicians, scientific theories can be, and have been, inconsistent without thereby endangering the rationality of those who use them. The history of science contains sufficient evidence that proves that contradictions have been tolerated for long periods, and that during these periods, scientists have used efficiently their their inconsistent theories to describe, predict and even explain phenomena.

Considering the above, logicians and philosophers have developed three different types of research programs for the study of inconsistency in science:

Historical programs: this type of programs have a deeply descriptive approach to contradiction in science, "which concerns the question whether inconsistencies commonly appear in science, and whether scientists sometimes accept and reason from inconsistencies" (Šešelja 2017: 2).

Logical programs: these programs have a more "normative perspective, which concerns the questions whether we can rationally reason from an inconsistent set of premises without ending up in a logical explosion, and if so, how" (idem).

Methodological programs: this type of programs have "a normative perspective, which concerns the role of the standard of consistency in evaluations of scientific theories" (ibid).

In light of the problems discussed in Sec. 2.2, it has become clear that any attempt to understand the phenomenon of scientific inconsistency toleration must put these programs all together in some way. Meaning that, a satisfactorily approach to inconsistency toleration should allow for a way to understand how it is possible to reason from inconsistent information in science without arriving at arbitrary conclusions, it should also allow for some insights about the status of consistency in science and finally, it should help us to describe and explain accurately actual cases of inconsistency toleration in science (if any).

3.2 What is the best way to make things right?

The resulting problems of the abductive argument can be divided into two categories: those of the strength of the inferential link and those of the veracity and relevance of the information used in the argument. So, in order to prevent the same mistakes that are made when following the abductive argument, a serious change in the methodology is necessary. Such a change should take seriously the challenge of integrating historical, logical and methodological considerations.

Here, I propose to approach the phenomenon of inconsistency toleration from a different perspective, instead of trying to approach particular instance of inconsistency toleration with a paraconsistent toolbox, taking into account the epistemic causes of and solutions to the tolerated contradiction. When looking at the large majority of instances of inconsistency toleration, at the intersection of both causes and solutions of each case there is a common enemy: ignorance.

In fact, inconsistency toleration has been commonly explained by appealing to a certain type of ignorance: ignorance of truth values. When scientists find a contradiction in their theories, models, or fragments of their reasoning, they assume that, at least, one of the two components of the contradiction is false but they ignore which one (Laudan 1977: 56). In absence of consistent alternatives, and both elements of the contradiction being either equally reliable/useful, scientists are forced to tolerate contradiction while acquiring the knowledge that would suffice to dismiss any of the elements of the contradiction (Cf. Martínez-Ordaz 2020).

History of science has revealed that, the knowledge that scientists lack in cases of inconsistency toleration, generally, is *knowledge of the theoretical structure* of the fragment of their theory that causes the contradiction. This prevents scientists from determining which, if any, of the two mutually conflicting propositions is false. It is not until scientists grasp that fragment of the theory's inferential structure that they can determine whether the optimal reasoning within it is paraconsistent or not.

This is illustrated in Figure 1.

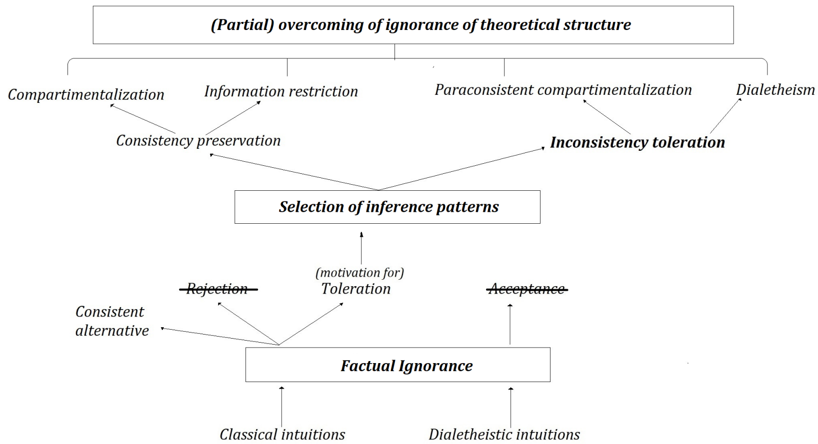

Figure 1. The general picture: ignorance and inconsistency toleration. In [Martínez-Ordaz 2020]

> The lowest level of the diagram illustrates what has already been said in Sect. 2: when ignoring the values of the conflicting propositions, scientists can either abandon the inconsistent set of information and work with a consistent alternative (if available), or be inclined to tolerate the contradiction temporarily.Footnote19 The middle area shows that, if scientists identify concrete inference patterns to work with the inconsistent information, depending on the type of inferential maneuvers that they chose, they can either preserve consistency or tolerate inconsistency. The highest level shows that, if ignorance of theoretical structure is (at least, partially) overcome, inconsistency toleration becomes available to the scientists; and also if inconsistency toleration has been achieved, this indicates the partial overcoming of ignorance of theoretical structure. (Martínez-Ordaz 2020)

To address the use of inconsistent information considering its connection with ignorance allows for suggests a methodological guide to investigate whether individual cases of tolerance to contradictions can in fact privilege any paraconsistent formal tool. The suggested path goes as follows:

1. Once a specific case of alleged temporal inconsistency toleration is identified, it is necessary to review it whitin its own context. This in order to distinguish whether it actually illustrates a conflict that should be interpreted as a contradiction and if there is no consistent alternative available.

   The questions that guide this first step include: which was the information that scientists did have access to? how was this information commonly segmented and combined? was there a domain that at the moment was thought to need the use of the inconsistent information in order to be described/measured/explained? was there any consistent alternative available?

2. The next step is to scrutinize the instance of inconsistency toleration in as much detail as possible, while seeking for the possible inference patterns that could allow working efficiently with the inconsistent fragment of the theory in that particular case.

   The questions that guide this first step include: Which were the procedures that scientists started avoided when the contradiction was noted? which are the most valuable outputs of using the inconsistent set of information in the discipline?

3. Once a set of inference patterns has been recognized, it is important to evaluate whether those patterns could have allowed the agents to optimally obtain the most important valuable inferential products of the theory (specially in terms of the use of computational resources).

4. Later on, it comes handy to classify the selected inference patterns considering whether they promote the *preservation of consistency* or *inconsistency toleration* in a paraconsistent sense (Cf. Bueno 2017). Here one should choose the interpretation that is most natural given the constraints of the particular case −methodological preferences of the discipline, doxastic agents, among others.

5. Only once all this research has been done, it will be possible to determine whether the case studied, in fact, constitutes evidence in favor of the use of any paraconsistent tools −and which, if any, is the tool that best allows us to reconstruct the reasoning led to the toleration of the inconsistency.

At this point, the reader might consider that this shift in methodological roads can be interesting but not necessarily would have an effect on our understanding of specific case studies, and more importantly, in our reading of specific cases in favor of certain paraconsistent logics. Yet, to believe such a thing would be a mistake.

In the next section, I scrutinize two different formal approaches to the study of inconsistency toleration in the sciences −one of which is logic oriented, while the other focuses on the detection of general inference patterns−, and argue that while the first is methodologically biased and very little informative, the second would be very revealing about both the phenomenon of inconsistency toleration in general and its paraconsistent component, if any.

4 Formal analyses of contradictions in science

In the previous section I have claimed that the study of inconsistency toleration in the sciences should combine historical and methodological programs with logical ones. And while I argued in favor of embracing formal approaches for the understanding of inconsistency toleration in scientific reasoning, I do not consider logical programs (as characterized in Sec. 3.1) by themselves to be robust enough for successfully accounting for the phenomenon of inconsistency toleration.

With this in mind, the section is divided in three parts. First, in Sec. 4.1., I discuss some of the problems that purely logical programs face when explaining cases of inconsistency toleration. Sec. 4.2. is devoted to present a peculiar sort of formal resources for the modeling of inconsistency toleration and in Sec. 4.3, I explain which are the features that formal tools should possess in order to promote the integration of methodological, historical and logical programs.

4.1 Logical programs and their problems

Logic understood from an epistemological point of view is mainly focused on increasing our understanding of human reasoning through the analyses of certain inferential patterns that agents could actually employ (Cf. Corcoran 1994). Such a view has provoked critical discussions on formal and philosophical levels.

On the one hand, some have argued that while it is not clear if logic could describe and norm always human inferential processes, it could still be explicative of *some* common inferences (Cf. Harman 1984). On the other hand, some others have contend that certain (type of) logics could ground a theory of human rationality. The latter approach consists in identifying a paradigmatic element of human rationality and analyzing the inferential patterns that are involved in them (which logical principles play a role in that particular type of reasoning, which are clearly avoided, and so on), the next step is to select a logic or a group of logics that can describe and explain such inferences. Ideally, the result of such analysis will provide us with, at least, a deeper understanding of human rationality (see Carnielli and Coniglio 2016: Chap.1).

Following a similar intuition, some schools of paraconsistent logics have persistently aimed at addressing under which circumstances certain logics can describe and explain and norm –actual- human reasoning in inconsistent contexts. Let's call this type of program the *Paraconsistent Logics Approach* (henceforth, *PLAs*).

The PLAs projects are mostly focused on the analysis of different types of logical consequence that could describe and explain sensible reasoning in inconsistent contexts −regardless if they are associated to scientific practices. As part of this approach one could recognize certain applications of Logic of Paradox (Cf. Priest 1984, 2006), some branches of the Adaptive Logics project (Cf. Batens 2002, 2017; Meheus 2002), some branches of the Logics of Formal Inconsistency (LFIs) project (Cf. Carnielli and Coniglio 2016: Chap. 8, 9), among others.

While this enterprise has produced many interesting formal results, it also has been accused of overlooking the actual phenomenon of handling inconsistency in scientific reasoning. This, partially because the type of analysis that PLAs holds requires strong commitments towards very specific (some of them even peculiar) logical consequence relations, which might not be part of human reasoning at all. In addition, so far these projects have not been able to agree on their explanations of which are the inferences that scientist could follow in order to avoid explosion when reasoning with inconsistent information; even more alarming, for the same case studies, different and rival explanations have been provided by the supporters of PLAs projects. And so far it has seem that either there is no core of shared elements that could explain how certain scientists have dealt with certain contradictions at a particular moment, or there are way too many alternative explanations that is no clear that any of those is actually an explanation for the particular cases.

In the majority of instances, the PLA-explanations of cases of inconsistency toleration are reinforced by specific applications of particular paraconsistent logics. And so, it has been argued that, PLAs draw the attention away from the actual premises and arguments offered by scientists by privileging discussions on which particular notion of logical consequence is, for instance, more virtuous (Brown and Priest, 2015).

For example, in [Meheus 2002] the case of Clausius' derivation of Carnot's theorem is presented as a case of inconsistent scientific reasoning, and it is explained by stating that the logic that satisfactorily models this type of reasoning is an Adaptive Logic, in particular, the adaptive logic ANA. The Clausius case is, methodologically, an interesting instance of PLAs because it consists of, first, the identification of a very specific piece of reasoning employed in the derivation of Carnot's theorem, and the further formal analysis of the (possible) logical constraints of the particular inferential path that was followed; the conclusion of the case study is that the reasoning that governed the derivation of the theorem was not only paraconsistent, but Adaptive, in the sense that after some inferential maneuvers that prevented logical explosion while working with inconsistent information, the reasoning went back to being explicitly classical. In light of this, Meheus proceeds to provide a logic, ANA, that can respect those constraints and lead still to the satisfactory derivation of the theorem. In this case, while the analysis is strongly *motivated* by the Adaptive Logics programme, and therefore it seems to be an instance of PLAs, the resulting logic ANA seems to be only inspired by the possibility of Clausius reasoning having been of an Adaptive spirit.

Another example of a PLAs-type of analysis can be found in Priest (1987, 2006) when he scrutinizes the physical phenomenon of motion as a contradictory one and provides an understanding of it that suits the basic structures of some dialetheist logics, at the ground of this proposal one can find the Hegelian theory of motion. It is important to note that this particular case is not as localized as the Clausius' one. On the contrary, Priest's approach is not about human reasoning, his analysis is expected to deal with one of the broadest physical phenomena and more importantly, it is expected to account for the most compelling explanation that was available in the early XX Century. Priest's analysis seems to conclude with the claim that the phenomenon of motion is a good instance of how dialetheias in the physical world might appear, which leads to accept that if this is the case, he has provided a physical domain that requires paraconsistent, and especially dialethiest logics, to be accounted for. An important note is that for the case of the Hegelian theory of motion, the resulting logic is not inspired by the phenomenon itself, but the phenomenon's characterization is inspired by the same philosophical intuitions that are placed behind dialetheism, so that the resulting logic is of a dialetheist spirit should not be surprising—yet, this might still be not problematic if the resulting formal description is successful.

Methodologically, Priest's ambitious project requires two main things to succeed: first, it should provide acceptable descriptions and explanations of the phenomenon that are compatible with the scientific findings available at the time; second, it should be more explanatory successful than any consistent rival available at the moment. Unfortunately, as it has been lengthy discussed in Boccardi and Macias-Bustos (2017), this is not the case. On the one hand, the Hegel-Priestian theory of motion has important explanatory and descriptive gaps, furthermore, the consistent alternative at the time, the Russellian theory of motion is explanatorily and descriptively more robust than the dialetheist. So, while the project seems interesting, the results are at best methodologically problematic.

The fact that PLA-explanations tend to privilege very specific logical consequences relations—of which further applications to scientific reasoning are not clear yet, makes less surprising that PLA-explanations face some harsh critiques from the history and philosophy of science—such as those that argue that the adoption of solely PLAs to historical episodes tends to threaten the understanding of the actual phenomenon.[5]

4.2 The Paraconsistent *Alternative* Approach

In face of this kind of allegations, a more general type of formal approach to inconsistency toleration has been suggested: general formal tools that do "not focus on identifying or proposing alternative logics that might lurk in the background of scientific reasoning. Instead it focuses on a more directly observable feature of reasoning, viz., how and where different premises are invoked in the course of arguments" (Brown and Priest 2015: 299). The result is a type of analysis of inconsistency in (scientific) reasoning through the use of some reasoning strategies; let's call this the *Paraconsistent Alternative Approach* (henceforth, *PAA*).

Considering that the PAA view makes no assumptions about which is the underlying logic of scientific reasoning, it is considered to be 'minimal' (Brown, 2017) when used to model specific cases from the history of sciences.

The PAA consists of a set of strategies or general procedures that are explanatory of the way in which it is possible to handle contradictions in order to avoid explosion. Such strategies are paraconsistent in the sense that they allow scientists to avoid logical explosion in an optimal way –recognizing that what is

[5] As it was claimed for the case of the hegelian theory of motion by Boccardi and Macías-Bustos (2017), and by Vickers (2013: 186-90) for some other interesting cases of alleged inconsistency toleration).

'optimal' would depend on the own constrains of each of the cases that are being studied. These strategies suggests ways in which information could be broken apart and transmitted while following some inferential patterns. Even though these strategies often substantiate the general dynamics of certain logics; they are, most of the time, also logic-independent –this is, they are compatible with many and diverse logical consequence relations.

Paraconsistent strategies do not necessarily focus on the structure of the scientific inconsistent theory (or model) itself, but they pay special attention to both, the information that epistemic agents often employ to identify the contradiction and the ways in which agents use such information in scientific problem solving and still avoid triviality. This minimal approach to inconsistent scientific reasoning was first sketched through the Rescher-Manor mechanisms (Rescher and Manor 1970) and is nowadays incarnated in the strategies that substantiate the dynamics of the so-called Adaptive Logics, reliability strategy and minimal abnormality strategy, among others (Cf. Verdeé 2009; Straßer 2014; Batens 2017), and in *Chunk and Permeate* (Cf. Brown 2016, 2017; Brown and Priest 2004, 2015; Friend 2013; Benham et al. 2014; Priest 2014).

In a similar way, another instance of the PAA could be identified in some of the works from the methodology of philosophy of science; in particular, the ones concerning the *Partial Structures Approach* (henceforth, PPEE) and the corresponding views on partial truth. PPEE consists on providing a methodology for the study of the ways in which scientific theories contain defective (partial, conflicting, inconsistent) information and still provide accurate predictions and explanations about target phenomena. The success of this approach is not motivated by any fixed commitments towards to a specific consequence relation, but it is actually caused by the fact that the PPEE's underlying structure is in a sense fragmentative with respect to, at least, conflicting sets of information. According to the PPEE, when working with a scientific theory, scientists are fully aware of the fact that not all the information that they have would hang together, some chunks of information would take as well known facts, others as well known falsities and some others are taken as deeply ignored elements. Yet nothing of this depends on any specific type of logical consequence relation, but it depends mostly on the ways in which we have seen scientists fill and update their theories.

There are two important remarks to make about the scope of PAA. First, as the PAA-strategies are more general and flexible, they allow for reconstructions that can incorporate more fine grained details from the historical record −satisfying the goals of some historical programs. But this virtue is not only about the expressive power of the formal apparatuses, as a matter of fact, the details that PAA allow for don't do an idle work; they help both philosophers and logicians to understand better why certain inferential procedures were drawn at certain moments, and why some others were avoided in each particular case. While this is informative of the inferential constraints of inconsistency toleration in the sciences, this would not be possible to grasp without adding some content (and context) to the analysis; which is something that the PLAs cannot do.

Second, because the PAA-strategies set the grounds for constructing logics −regardless whether they are paraconsistent or not−, the analyses that result from applaying PAAs to particular cases, (can) end up being about the logical consequence relation that was built in each case. Yet, the way through which this is obtained is methodologically much more virtuous than the paths that are followed when using the PLAs. In particular, as the application of PAAs results from shifting the attention from initial philosophical concerns about specific privileged logical consequence relations to concrete flows of information and inference patterns in concrete cases; if any conclusion is reached according to which the logic that was built through the development of the case is paraconsistent, such a conclusion is free of methodological biases like the abductive leap (Sec. 2.2).

In what follows, I explore very, briefly, an interesting type of logical pluralism that emerges from the use of PAA.

5 The pluralism that comes with PAAs

One of the main concerns that someone might have towards the efficiency of PAAs is how to deal with the underdetermination of theory by data that would natural emerge from the setup of PAAs. The main idea behind this concern is that the evidence available to us that favors a reasoning strategy over its rivals at a given time, may be insufficient to determine what beliefs with respect to specific logics we should hold in response to such an evidence. In this last section, I address this issue and explain that the referring undetermination provides the grounds of a peculiar type of logical pluralism.[6]

Logical pluralism is often seen as the acceptance of the fact that "different kinds of situations, and different logics (or consequence relations) may be appropriate for reasoning about them—in the sense that if you know (or assume) that certain things hold in these situations, the logic is guaranteed to give you other things that hold in the situation" (Priest 2015: 331). All this in the absence of the assumption that a single correct approach to the reasoning exists.

As the PAA are mostly interested in analyzing general procedures that help to attain reliable information through the use of inconsistent data, and because these reasoning strategies are -most of the time- also logic-independent −in the sense of being compatible with different logical consequence relations. This allows for a sort of logical pluralism.

> **PAA-Logical Pluralism:** In absence of the assumption that a single correct approach to the logical consequence that underlies the inconsistency toleration processes exists, a PAA-view suggests that there is a finite number of equally successful inference strategies that help to handle contradictions in human reasoning.

[6] I am indebted to Moisés Macías-Bustos who helped to give a better phrasing of my ideas on this point.

> While these strategies guide very general procedures of information management, such as to separate the information in maximally consistent sets, or to distrust certain type of results, they are also compatible with different non arbitrary types of logical consequence relations; and there is where the secret of their explanatory and descriptive power lies. As a matter of fact, the PAA is in the large majority of cases accurate because it allows for a unique openness regarding the procedures and the corresponding interpretations of human reasoning.

One could fear that this logical pluralism is actually a type of logical relativism; this is the view that there can be as many correct logics as inferential subjects (either individuals or collectives) (Baghramian and Carter 2018: Sec. 4.4). This, of course could diminish the normative component that, intuitively, logic possess. However, I think, that this might not be the case, as, while all PAA could be abstractly compatible with extremely many and diverse logics, when being employed to model specific cases of scientific reasoning, only some of them would be relevant and suitable for doing this job –depending on the particular cases to be modeled.

In addition, even if the different strategies that could be used for formally reconstructing specific cases of inconsistency toleration provide different reconstructions of the same phenomenon, it is very likely that such reconstructions reveal different components of the same episode of scientific rationality and while doing so, enrich our understanding of both, the reasoning that took place at that particular moment as well as the general phenomenon of scientific rationality. Finally, considering that the PAA takes into account general elements involved in the most common practices of inconsistency toleration in science, this approach also permits that, depending on the particularities of each case of inconsistency toleration, different logics are used in order to transmit, select and neglect certain type of information. In order to provide a successful approach to inconsistency toleration a paraconsistent reasoning strategy should describe the use of the most natural information-transmitting inferences into, at least, conditional operations (Cf. Meheus 2002), and if doing so, a PAA could, in the long run, shed light on the basic elements of human rationality in inconsistent contexts.

6 Final remarks

Inconsistency toleration is a phenomenon that takes place once epistemic agents who, after identifying a contradiction, can still reason sensibly from it. And while it is possible to spot many instances of inconsistency toleration in the history of science, it constitutes a huge methodological mistake to interpret such instances as direct evidential support in favor of the success or legitimacy of any paraconsistent logic.

With this methodological problem in mind, here I proposed to shift the analysis of cases of inconsistency toleration from seeking for hits about logical consequence relations to paying special attention to both the epistemological causes of the toleration of a contradiction and the overcoming of the problem. I explained that, in the large majority of cases, inconsistency toleration is triggered by a particular type of ignorance, *ignorance of theoretical structure*, this is, ignorance of the inference patterns that hold within an inconsistent set of information. The overcoming of such an ignorance is gradual and consists of the identification of privileged reasoning strategies that allow agents to work with inconsistent data and not harm their rationality when doing so. I contended that in order to understand the phenomenon of inconsistency toleration (at an inferential level), it was necessary to scrutinize the adoption of these inference strategies considering the epistemological particularities of each particular case.

Furthermore, I considered two different methodologies used when scrutinizing cases of inconsistency toleration in the sciences. The first one was the *Paraconsistent Logics Approach* and the second one, the *Paraconsistent Alternative Approach*. After explaining each of them, I argued that, while the former faces many difficulties (one of them being the fallacious abductive leap, and another, the overlook of the actual phenomenon of inconsistency toleration), the latter could, through a type of pluralism, maintain the virtues of the Paraconsistent Logics approach and at the same time, give account of the particularities of each case of inconsistent reasoning in science.

It is important to notice that, when making these methodological changes, many of the alleged instances of inconsistency toleration through paraconsistency might also shift to be characterized as only apparent or as results of consistency preserving (and non-paraconsistent) maneuvers; however, this apparent loss would be a methodological victory over the challenges of the abductive argument. In sum, this methodological shift would be only in favor of *some* better justified paraconsistency in the sciences.

Acknowledgments:

This research was supported by the Programa Nacional de Pós-Doutorado PNPD CAPES (Brazil).
I am indebted to Moisés Macías-Bustos, Mary Gwin, Jonas Arenhart, Otavio Bueno, Gabrielle Ramos-Garcia and Itala Loffredo D'Ottaviano for fruitful discussions on these issues. I owe special thanks to the anonymous referees for their valuable comments and suggestions. Thanks to the audience at *The Heterodox in Logic and Reason (WLD 2021).*

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